Proving similarity

Similar figures are shapes that have same shape but different in size.

Two shapes are said to be similar if :

1. All corresponding angles are equal.

2. All corresponding sides are in ratio.

To prove that two Geometric figures are similar, the following critereon should be in proportion.

(a) All corresponding angles should be equal.

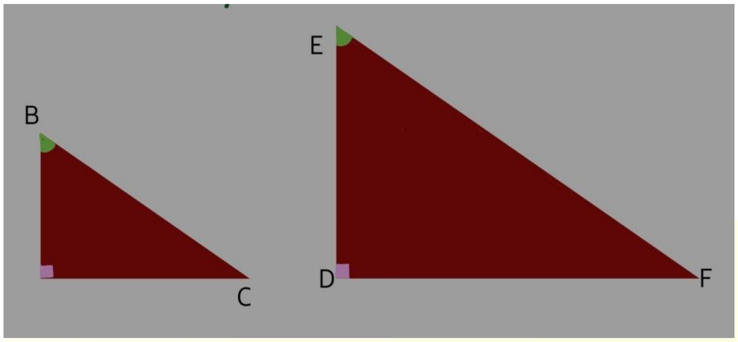
(b) All corresponding sides should be in proportion.

Proving Similarity in triangles –

For Triangles, the following critereon are commonly used to establish similarity.

(a) Angle – Angle (AA) – If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

This is because the third angles will aslo be equal , satisfying the angle sum property of triangles.



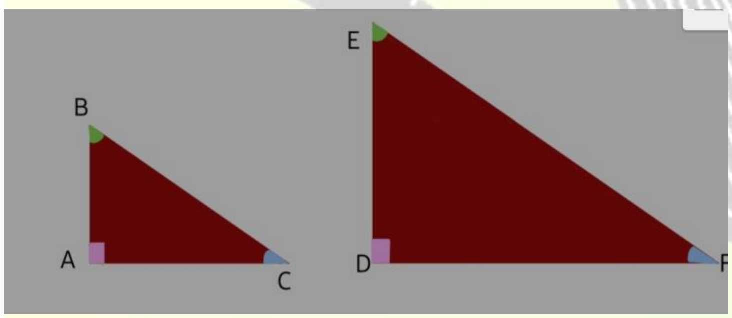
In the figure given above

∠ABC = ∠DEF

∠BAC = ∠EDF

By AA similarity, they are

(b) Angle Angle Angle (AAA) : If all 3 corresponding angles of two triangles are equal then the triangles are similar.



In triangles given above –

∠ABC = ∠DEF

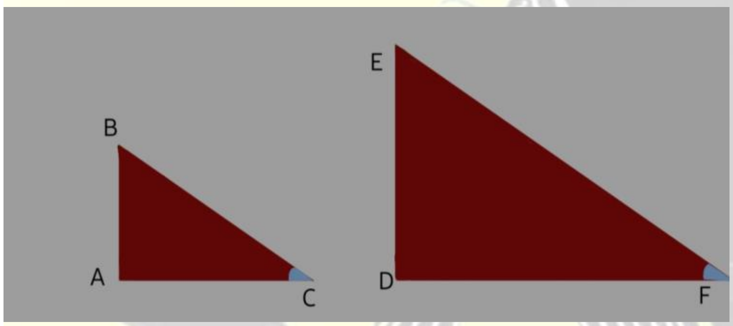
∠BAC = ∠EDF

∠BCA = ∠EFD

By AAA similarity these two triangles are similar.

(c) Side angle side (SAS) similarity –

If one angle of a triangle is equal to the corresponding angle of another triangle and corresponding sides including these angles are in same proportion then the triangles are similar.



In the triangles given above

$\angle BCA = \angle EFD$

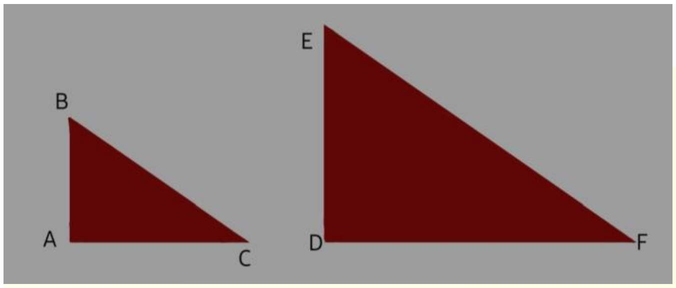
$\frac{AC}{DF} = \frac{BC}{EF} = k$

$\angle C = \angle F$

By SAS similarity above two triangle are similar.

(d) Side Side Side similarity –

If all the corresponding sides of two triangles are in same proportion, then the triangles are similar.

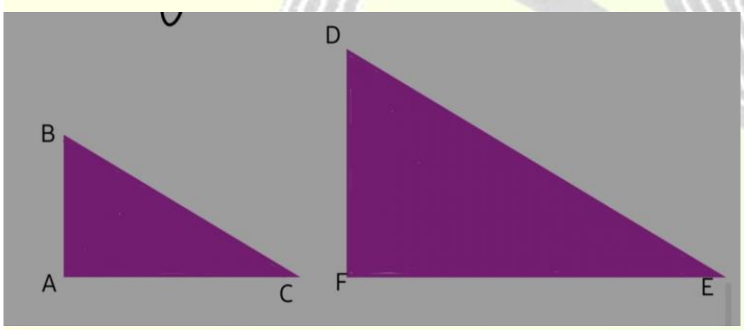


In triangles given above

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{FD}{CA} = k$$

Hence they both are similar by side side side similarity.

Q.1 There are two right triangles given in the figures.



If AB = 4 cm, AC = 10 cm, DF = 12 cm, FE = 30 cm, Then prove both triangles to be similar.

Sol – In the triangle given above –

$\frac{AB}{DF} = \frac{4}{12} = \frac{1}{3}$

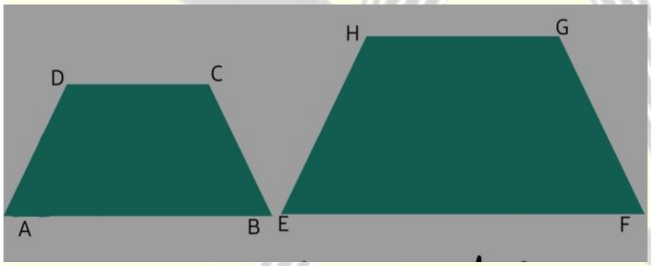
$\frac{AC}{FE} = \frac{10}{30} = \frac{1}{3}$

Hence two sides are in proportion

∠A = ∠F = 90^0

Therefore by SAS Similarity both triangles are similar.

Q.2



In the figure given above ∠A = 80^0, ∠C = 120^0, ∠H = 100^0, ∠F = 60^0, Then state whether these two trapezium are similar or not.

Sol – In above trapezium

∠A = 80^0

∠D = 180^0 - 80^0

$= 100^0$ (as DC and AB are parallel ∠D and ∠A are interior angles).

$\angle C = 120^0$

$\angle B = 180 - 120$

$= 60^0$ (∠C and ∠B are interior angles)

$\angle H = 100^0$

$\angle E = 180^0 - 100^0$

$= 80^0$ (HG and EF are parallel lines. ∠H and ∠E are interior angles)

$\angle F = 60^0$

$\angle G = 180^0 - 60^0$

$= 120^0$ (∠F and ∠G are interior angles)

Hence,

$\angle A = \angle E$

$\angle D = \angle H$

$\angle C = \angle G$

$\angle B = \angle F$

As all corresponding angles are equal. Hence these two trapezium are similar.